

# Dynamical Symmetries in Supersymmetric Matrix Models\*

V. Bach,<sup>1</sup> J. Hoppe,<sup>2</sup> D. Lundholm<sup>3</sup>

## Abstract

We reveal a dynamical  $SU(2)$  symmetry in the asymptotic description of supersymmetric matrix models. We also consider a recursive approach for determining the ground state, and point out some additional properties of the model(s).

## 1 Introduction

Ten years ago, a lot of effort was put into the question of zero-energy states in  $SU(N)$ -invariant supersymmetric matrix models. While the attempt to explicitly construct such a state mainly [1] used a space-independent (but manifest  $SO(d)$ -invariance-breaking) decomposition of the fermions into creation and annihilation operators, the asymptotic form of the wave function was determined with the help of space-dependent fermions [2, 3, 4].

In this paper we would like to point out a dynamical  $SU(2)$  symmetry and the formation of ‘Cooper pairs’ that arise in the  $SO(d)$ -breaking formulation when diagonalizing certain ingredients of the fermionic part of the Hamiltonian. We start by considering the asymptotic  $SU(2)$  theory but note that several features extend to the non-asymptotic and general  $SU(N)$  cases.

## 2 Asymptotic form of the Hamiltonian

The bosonic configuration space is a set of  $d = 2, 3, 5$ , or  $9$  traceless hermitian matrices  $X_s$ , corresponding to the Lie algebra of the gauge group  $SU(N)$ . For simplicity, we start by taking  $N = 2$ . Coordinatizing regions where the bosonic potential,

$$V = -\frac{1}{2} \sum_{s,t=1}^d \text{tr} [X_s, X_t]^2, \quad (1)$$

is zero by (cp. e.g. [3, 4])

$$\begin{aligned} X_t &= r \cos \theta \tilde{E}_t \left( \frac{1}{2} e_A \sigma_A \right), \quad t = 1, \dots, d-2 \\ X_{d-1} + iX_d &= r \sin \theta e^{i\varphi} \left( \frac{1}{2} e_A \sigma_A \right), \end{aligned} \quad (2)$$

---

\*Supported by the Swedish Research Council

<sup>1</sup>vbach@mathematik.uni-mainz.de

<sup>2</sup>hoppe@math.kth.se

<sup>3</sup>dogge@math.kth.se

where  $e^2 = 1$ ,  $\sum_{t=1}^{d-2} \tilde{E}_t^2 = 1$  and  $\sigma_A$  are the Pauli matrices,  $\frac{1}{r}$  times the effective asymptotic Hamiltonian (cf. [4, 1]) becomes

$$\begin{aligned} H^\infty = H_B^\infty &+ 2 \cos \theta (-ie_C \epsilon_{ABC}) \Gamma_{\alpha\beta} \lambda_{\alpha A} \partial_{\lambda_{\beta B}} \\ &+ \sin \theta e^{i\varphi} (e_C \epsilon_{ABC}) \lambda_{\alpha A} \lambda_{\alpha B} \\ &+ \sin \theta e^{-i\varphi} (e_C \epsilon_{ABC}) \partial_{\lambda_{\alpha B}} \partial_{\lambda_{\alpha A}}. \end{aligned} \quad (3)$$

The last three terms are the leading ones in the fermionic part of the Hamiltonian (as  $r \rightarrow \infty$ ) while  $H_B^\infty$ , which arises from  $-\Delta + V$  in that limit, denotes an independent set of harmonic oscillators on  $\mathbb{R}^{s_d}$  with ground state energy  $s_d = 2, 4, 8$ , or  $16$ .  $\Gamma := \sum_{t=1}^{d-2} \tilde{E}_t \Gamma_t$  is a purely imaginary, antisymmetric, and hence self-adjoint  $\frac{s_d}{2} \times \frac{s_d}{2}$ -matrix squaring to unity.  $\lambda_{\alpha A}$  and  $\partial_{\lambda_{\alpha A}} = \lambda_{\alpha A}^\dagger$  are space-independent fermion creation resp. annihilation operators satisfying

$$\begin{aligned} \{\lambda_{\alpha A}, \partial_{\lambda_{\beta B}}\} &= \delta_{\alpha\beta} \delta_{AB}, \\ \{\lambda_{\alpha A}, \lambda_{\beta B}\} &= \{\partial_{\lambda_{\alpha A}}, \partial_{\lambda_{\beta B}}\} = 0 \end{aligned} \quad (4)$$

and acting on the fermionic vacuum state  $|0\rangle$ , defined by  $\partial_{\lambda_{\alpha A}} |0\rangle = 0 \ \forall \alpha, A$ .

We define space-dependent fermion creation operators (for  $d = 3, 5, 9$ )

$$\lambda_{\sigma j \tau} := (\tilde{e}_{\sigma j})_\alpha (\mathbf{n}_\tau)_A \lambda_{\alpha A}, \quad (5)$$

where  $\sigma, \tau$  denote  $+$  or  $-$ , and  $\mathbf{n}_\pm \in \mathbb{C}^3$  resp.  $\tilde{e}_{\sigma j} \in \mathbb{C}^{s_d/2}$  are eigenvectors of  $(-ie_C \epsilon_{CAB})$  resp.  $\Gamma_{\alpha\beta}$ ,

$$i\mathbf{e} \times \mathbf{n}_\pm = \pm \mathbf{n}_\pm \quad (6)$$

$$\Gamma \tilde{e}_{\pm j} = \pm \tilde{e}_{\pm j}, \quad j = 1, \dots, \frac{s_d}{4}. \quad (7)$$

We choose these to depend continuously on  $\mathbf{e}$  and  $\tilde{E}$ , as well as to be orthonormal and such that the complex conjugates  $(\mathbf{n}_\pm)^* = \mathbf{n}_\mp$  and  $(\tilde{e}_{\pm j})^* = \tilde{e}_{\mp j}$ . The asymptotic Hamiltonian  $H^\infty$ , when acting on the ground state of  $H_B^\infty$ , can then be written as

$$H = H_0 + H_+ + H_-, \quad (8)$$

$$\begin{aligned} H_0 &= s_d + 2 \cos \theta \sum_j (N_{A_j} - N_{B_j}), \\ H_+ &= 2 \sin \theta \sum_j (A_j + B_j), \\ H_- &= 2 \sin \theta \sum_j (A_j^\dagger + B_j^\dagger), \end{aligned} \quad (9)$$

where

$$\begin{aligned} A_j &:= ie^{i\varphi} \lambda_{+j+} \lambda_{-j-}, \\ B_j &:= ie^{i\varphi} \lambda_{-j+} \lambda_{+j-}, \end{aligned} \quad (10)$$

satisfy

$$\begin{aligned} [A_j, A_j^\dagger] &= N_{A_j} - 1 := \lambda_{+j+} \partial_{\lambda_{+j+}} + \lambda_{-j-} \partial_{\lambda_{-j-}} - 1, \\ [B_j, B_j^\dagger] &= N_{B_j} - 1 := \lambda_{-j+} \partial_{\lambda_{-j+}} + \lambda_{+j-} \partial_{\lambda_{+j-}} - 1. \end{aligned} \quad (11)$$

For the  $d=2$  case we instead of (5) define  $\lambda_\pm := (\mathbf{n}_\pm)_A \lambda_A$  and the corresponding expressions for the asymptotic Hamiltonian are simply

$$H_0 = 2, \quad H_+ = 2C, \quad H_- = 2C^\dagger, \quad C := ie^{i\varphi} \lambda_+ \lambda_- \quad (12)$$

### 3 Dynamical symmetry

Let us now restrict to  $d=9$  (for definiteness). Denoting  $A := \sum_j A_j$  by  $J_+ \otimes 1$ ,  $A^\dagger = \sum_j A_j^\dagger$  by  $J_- \otimes 1$ ,  $\frac{1}{2}(N_A - 4) := \frac{1}{2}(\sum_j N_{A_j} - 4)$  by  $J_3 \otimes 1$ , and similarly  $1 \otimes J_+$ ,  $1 \otimes J_-$ ,  $1 \otimes J_3$  for the  $B$ s, with

$$[J_+, J_-] = 2J_3, \quad [J_3, J_\pm] = \pm J_\pm, \quad J_\pm = J_1 \pm iJ_2, \quad (13)$$

eqs. (8), (9) can be written as

$$\frac{1}{4}H = (2 + \cos \theta J_3 + \sin \theta J_1) \otimes 1 + 1 \otimes (2 - \cos \theta J_3 + \sin \theta J_1), \quad (14)$$

thus exhibiting the dynamical symmetry mentioned above. The relevant  $SU(2)$  representations are the tensor product of four spin  $\frac{1}{2}$  representations, i.e. direct sums of two singlets (note that both  $(A_1 A_3 + A_2 A_4 - A_1 A_4 - A_2 A_3)|0\rangle$  and  $(A_1 A_2 + A_3 A_4 - A_1 A_4 - A_2 A_3)|0\rangle$  are annihilated by  $A$ ,  $A^\dagger$ , and  $\frac{1}{2}(N_A - 4)$ ), three spin 1 representations, and (most importantly, as providing the zero-energy state of  $H$ ) one spin 2 representation acting irreducibly on the space spanned by the orthonormal states

$$|0\rangle, \quad \frac{1}{2}A|0\rangle, \quad \frac{1}{\sqrt{24}}A^2|0\rangle, \quad \frac{1}{12}A^3|0\rangle, \quad \frac{1}{4!}A^4|0\rangle. \quad (15)$$

Restricting to that space (correspondingly for the  $B$ s), we can write

$$J_3 = \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}, \quad J_+ = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}, \quad J_- = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (16)$$

Since the spectrum of  $\sin \theta J_1 \pm \cos \theta J_3$  is the same as that of  $J_3$ , the spectrum of  $\frac{1}{4}H$  clearly consists of all integers between zero and eight, with the unique zero-energy state  $\Psi$  most easily obtained by solving individually, for each  $A_j$  resp.  $B_j$  degree of freedom,

$$\left(1 \pm \cos \theta \sigma_3^{(j)} + \sin \theta \sigma_1^{(j)}\right) \Psi = e^{\mp \frac{1}{2} \theta i \sigma_2^{(j)}} \left(1 \pm \sigma_3^{(j)}\right) e^{\pm \frac{1}{2} \theta i \sigma_2^{(j)}} \Psi \stackrel{!}{=} 0, \quad (17)$$

where we identify  $2J_k = \sigma_k^{(1)} \otimes 1 \otimes 1 \otimes 1 + \dots + 1 \otimes 1 \otimes 1 \otimes \sigma_k^{(4)} = \sum_{j=1}^4 \sigma_k^{(j)}$ . In our notation  $\sigma_3^{(j)}|0\rangle = -|0\rangle$  and  $\sigma_3^{(j)}A_j|0\rangle = +A_j|0\rangle$ , and we easily find the solution to (17) as

$$\Psi = \left(\prod_j e^{-\frac{\theta}{2} i \sigma_2^{(j)}}\right) \left(\prod_j e^{\frac{\theta}{2} i \sigma_2^{(j)}} B_j\right) |0\rangle = e^{-\theta i (J_2 \otimes 1 - 1 \otimes J_2)} \frac{B^4}{4!} |0\rangle. \quad (18)$$

Using the nilpotency of  $A_j$  and  $B_j$  for

$$e^{\alpha(A_j - A_j^\dagger)}|0\rangle = \cos \alpha e^{\tan \alpha A_j}|0\rangle \quad \text{and} \quad e^{-\alpha(B_j - B_j^\dagger)}|0\rangle = \sin \alpha e^{\cot \alpha B_j}|0\rangle, \quad (19)$$

the ground state can also be written as

$$\begin{aligned} \Psi &= \frac{1}{16} e^{-4i\varphi} (\sin \theta)^{-4} \prod_j (\sin \theta - (1 - \cos \theta) A_j) (\sin \theta - (1 + \cos \theta) B_j) |0\rangle \\ &= \frac{1}{16} e^{-4i\varphi} (\sin \theta)^4 e^{-\frac{1 - \cos \theta}{\sin \theta} A - \frac{1 + \cos \theta}{\sin \theta} B} |0\rangle \sim e^{-C_\theta} |0\rangle, \end{aligned} \quad (20)$$

with  $C_\theta := \frac{1-\cos\theta}{\sin\theta}(J_+ \otimes 1) + \frac{1+\cos\theta}{\sin\theta}(1 \otimes J_+)$ . Alternatively, one can solve the  $2 \times 2$  matrix eigenvector equations resulting from (17),

$$\begin{aligned} \left(1 + \cos\theta(N_{A_j} - 1) + \sin\theta(A_j + A_j^\dagger)\right) \Psi &= 0, \\ \left(1 - \cos\theta(N_{B_j} - 1) + \sin\theta(B_j + B_j^\dagger)\right) \Psi &= 0, \end{aligned} \quad (21)$$

to obtain (20).

For  $d=2$  the asymptotic ground state is easily found from (12),

$$\Psi = \frac{1}{\sqrt{2}} e^{-C} |0\rangle = \frac{1}{\sqrt{2}} (1 - C) |0\rangle. \quad (22)$$

An interesting feature of the form (18) for the ground state is that it expresses it as a spin-rotation by an angle  $\theta$  applied to some reference state  $B^4|0\rangle$  (which itself also varies in the first  $d-2$  directions in space according to (2), (5), (7)).

## 4 Graded chain of Hamiltonians

Consider the grade- resp. fermion number-ordered equations

$$\begin{aligned} H_0 \Psi_0 + H_- \Psi_2 &= 0, \\ H_+ \Psi_0 + H_0 \Psi_2 + H_- \Psi_4 &= 0, \\ H_+ \Psi_2 + H_0 \Psi_4 + H_- \Psi_8 &= 0, \\ &\vdots \\ H_+ \Psi_{12} + H_0 \Psi_{14} + H_- \Psi_{16} &= 0, \\ H_+ \Psi_{14} + H_0 \Psi_{16} &= 0, \end{aligned} \quad (23)$$

implied by  $H\Psi = (H_0 + H_+ + H_-)(\Psi_0 + \Psi_2 + \dots + \Psi_{16}) \stackrel{!}{=} 0$ . (We have dropped the eight non-dynamical parallel fermions  $\lambda_\alpha^\parallel := \lambda_{\alpha A} e_A$ .) The following method to construct the ground state we believe to be relevant also for the fully interacting, non-asymptotic theory. Use the first equation in (23) to express  $\Psi_0$  in terms of  $\Psi_2$ ,

$$\Psi_0 = -H_0^{-1} H_- \Psi_2. \quad (24)$$

$H_0$  is certainly invertible on the zero-fermion subspace, even in the full theory, where (cf. [1])

$$H_0 = -\Delta + V - 2ix_{jC} f_{CAB} \Gamma_{\alpha\beta}^j \lambda_{\alpha A} \lambda_{\beta B}^\dagger. \quad (25)$$

Using (24), the second equation in (23) can be written as

$$H_2 \Psi_2 + H_- \Psi_4 = 0, \quad \text{with} \quad H_2 := H_0 - H_+ H_0^{-1} H_-, \quad (26)$$

yielding

$$\Psi_2 = -H_2^{-1} H_- \Psi_4, \quad (27)$$

provided  $H_2$  is invertible on  $H_- \Psi_4$ , resp. the two-fermion sector of the Hilbert space. Continuing in this manner, denoting

$$\hat{\mathcal{H}}_{2k} := \text{Span}\{A^m B^n |0\rangle\}_{m,n=0,1,2,3,4, \ m+n=k} \quad (28)$$

for the considered  $2k$ -fermion subspace, we find that if we assume invertibility of  $H_{2k}$  on  $\mathcal{H}_{2k}$  we can form

$$H_{2(k+1)} := H_0 - H_+ H_{2k}^{-1} H_- \quad (29)$$

on  $\hat{\mathcal{H}}_{2(k+1)}$  and solve for  $\Psi_{2k}$  in terms of  $\Psi_{2(k+1)}$ . The final equation for  $\Psi_{16}$  is  $H_{16}\Psi_{16} = 0$ .

For concreteness, denote an orthonormal basis of  $\hat{\mathcal{H}} = \oplus_k \hat{\mathcal{H}}_{2k}$  by  $|k, l\rangle := |k\rangle \otimes |l\rangle$ , where, as in (15),

$$|k\rangle := \frac{1}{k! \sqrt{\binom{4}{k}}} J_+^k |0\rangle. \quad (30)$$

Then  $H_+ H_0^{-1} H_-$ , e.g., acts on  $\hat{\mathcal{H}}$  ‘tridiagonally’ according to

$$\begin{aligned} \frac{1}{\sin^2 \theta} H_+ H_0^{-1} H_- |k, l\rangle &= \left( \frac{k(5-k)}{4+(k-l-1)\cos\theta} + \frac{l(5-l)}{4+(k-l+1)\cos\theta} \right) |k, l\rangle \\ &+ \frac{\sqrt{l(5-l)(k+1)(4-k)}}{4+(k-l+1)\cos\theta} |k+1, l-1\rangle \\ &+ \frac{\sqrt{k(5-k)(l+1)(4-l)}}{4+(k-l-1)\cos\theta} |k-1, l+1\rangle. \end{aligned} \quad (31)$$

Calculating the spectra of  $H_{2k}$  on  $\hat{\mathcal{H}}_{2k}$  (e.g. with the help of a computer) one can verify the invertibility of all  $H_{2k}$  on  $\hat{\mathcal{H}}_{2k}$  for  $k < 8$ , while  $H_{16}$  is identically zero on  $\hat{\mathcal{H}}_{16}$ . Hence, one can also start with the state  $\Psi_{16} \sim A^4 B^4 |0\rangle$  (with correct normalization in  $\theta$ ) and generate the lower grade parts of the full ground state  $\Psi$  using the relations (24), (27), etc.

Let us finish this section by noting a simple consequence of the graded form (23) of the ground state equation  $H\Psi = 0$  (for general  $d$  and  $N$ ). Taking the inner product of the grade  $2k$ -equation with  $\Psi_{2k}$  yields

$$\begin{aligned} \langle \Psi_{2k}, H_- \Psi_{2(k+1)} \rangle &= -\langle H_0 \rangle_{2k} - \langle \Psi_{2k}, H_+ \Psi_{2(k-1)} \rangle \\ &= -\langle H_0 \rangle_{2k} - \langle \Psi_{2(k-1)}, H_- \Psi_{2k} \rangle^*, \end{aligned} \quad (32)$$

where  $\langle H_0 \rangle_{2k} := \langle \Psi_{2k}, H_0 \Psi_{2k} \rangle$ . The first equation reads  $\langle \Psi_0, H_- \Psi_2 \rangle = -\langle H_0 \rangle_0$  which is real. The second then becomes  $\langle \Psi_2, H_- \Psi_4 \rangle = -\langle H_0 \rangle_2 + \langle H_0 \rangle_0$ , and so on, so that in the last step one obtains

$$\sum_{k=0}^{\Lambda} (-1)^k \langle H_0 \rangle_{2k} = 0, \quad (33)$$

where  $\Lambda$  is the total number of fermions in the relevant Fock space.

It is instructive to verify (33) for the asymptotic  $N = 2$  case studied above, since there all relevant terms can be calculated explicitly. Using the basis (30) and the notation  $\alpha := 1 - \cos\theta$ ,  $\beta := 1 + \cos\theta$ , we find

$$\Psi \sim e^{-C_\theta} |0\rangle = \sum_k \frac{(-1)^k \sqrt{\binom{4}{k}}}{(\sin\theta)^k} \alpha^k |k\rangle \otimes \sum_l \frac{(-1)^l \sqrt{\binom{4}{l}}}{(\sin\theta)^l} \beta^l |l\rangle. \quad (34)$$

Hence,

$$\langle \Psi_{2n}, H_0 \Psi_{2n} \rangle = \frac{1}{64} (\sin\theta)^{8-2n} \sum_{k+l=n} \binom{4}{k} \binom{4}{l} (4 + (k-l)\cos\theta) \alpha^{2k} \beta^{2l}. \quad (35)$$

## 5 General SU( $N$ )

Let us now derive, for general  $N \geq 2$ , the ground state energy of

$$H_F = i\gamma_{\alpha\beta}^t f_{ABC} x_{tC} \theta_{\alpha A} \theta_{\beta B} \quad (36)$$

in regions of the configuration space where the potential  $V$  is zero. (As in (25),  $f_{ABC}$  denote the structure constants of SU( $N$ ) in an orthonormal basis.) By (1) this means that all  $X_s$  are commuting, hence can be written  $X_s = U D_s U^\dagger$  where  $U$  is unitary and independent of  $s$  and the  $D_s$  are diagonal. If we look into a particular direction (corresponds to fixing  $e$  in the SU(2) case) and choose a basis  $\{T_A\}$  accordingly we may write  $X_s = D_s = x_{sA} T_A = x_{s\tilde{k}} T_{\tilde{k}}$  and  $x_{sa} = 0$ , where  $\tilde{k} = 1, \dots, N-1$  are indices in the Cartan subalgebra and  $a, b = N, \dots, N^2-1$  denote the remaining indices.

Denoting the eigenvalues of  $X_t$  by  $\mu_k^t$ , i.e.  $X_t = \text{diag}(\mu_1^t, \dots, \mu_N^t)$ , then the eigenvectors  $\{e_{kl}\}_{k \neq l}$  of  $M_{ab}^t := -i f_{abC} x_{tC} = -i f_{ab\tilde{k}} x_{t\tilde{k}}$  satisfy (cf. e.g. [5])

$$M^t e_{kl} = (\mu_k^t - \mu_l^t) e_{kl} =: \mu_{kl}^t e_{kl}, \quad (e_{kl}^a)^* = e_{lk}^a. \quad (37)$$

The crucial observation is that these eigenvectors are independent of  $t$ . Now,

$$H_F = -\gamma_{\alpha\beta}^t M_{ab}^t \theta_{\alpha a} \theta_{\beta b} = W_{\alpha a, \beta b} \theta_{\alpha a} \theta_{\beta b}, \quad (38)$$

where  $W := -\sum_t \gamma^t \otimes M^t$ . From the above observations we have the ansatz  $E_{\mu kl} := v_\mu \otimes e_{kl}$  for the eigenvectors of  $W$ , giving

$$W E_{\mu kl} = -\sum_t \gamma^t v \otimes M^t e_{kl} = \gamma(k, l) v_\mu \otimes e_{kl}, \quad (39)$$

where  $\gamma(k, l) := -\sum_t \mu_{kl}^t \gamma^t$  squares to  $\sum_t (\mu_{kl}^t)^2$ . Letting  $v_\mu = v_{\pm jkl}$  denote the corresponding 16 eigenvectors of  $\gamma(k, l)$ , we find

$$W E_{\pm jkl} = \pm \sqrt{\sum_t (\mu_{kl}^t)^2} E_{\pm jkl} \quad (40)$$

and  $H_F$  therefore has  $E_0 := -16 \sum_{k < l} \sqrt{\sum_{t=1}^9 (\mu_k^t - \mu_l^t)^2}$  as its lowest eigenvalue.

This agrees with the following two previously known cases: [5], where only  $X_9$  is assumed to have large eigenvalues so that  $E_0 \rightarrow -16 \sum_{k < l} |\mu_k^9 - \mu_l^9|$ ; as well as the SU(2)-case studied above, where (2) with e.g.  $e_A = \delta_{A3}$  gives  $E_0 = -16r$ .

## Acknowledgements

One of us (J.H.) would like to thank Choonkyu Lee for his hospitality, and Ki-Myeong Lee for useful discussions.

## References

- [1] J. Hoppe, *On the construction of zero energy states in supersymmetric matrix models I, II, III*, hep-th/9709132, 9709217, 9711033.

- [2] M.B. Halpern, C. Schwartz, *Asymptotic search for ground states of  $SU(2)$  matrix theory*. Int. J. Mod. Phys. A 13 (1998) 4367, hep-th/9712133.
- [3] G.M. Graf, J. Hoppe, *Asymptotic ground state for 10-dimensional reduced supersymmetric  $SU(2)$  Yang-Mills theory*, hep-th/9805080.
- [4] J. Fröhlich, G.M. Graf, D. Hasler, J. Hoppe, S.-T. Yau, *Asymptotic form of zero energy wave functions in supersymmetric matrix models*, Nucl. Phys B 567 (2000), 231-248.
- [5] D. Hasler, J. Hoppe, *Asymptotic factorisation of the ground-state for  $SU(N)$ -invariant supersymmetric matrix-models*, hep-th/0206043.

V. Bach, FB Mathematik, Universität Mainz, DE-55099 Mainz, Germany

J. Hoppe and D. Lundholm, Department of Mathematics, Royal Institute of Technology, SE-10044 Stockholm, Sweden